

GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES
TO FIND THE SOLUTION FOR A GRAPHICAL LOGISTICAL MODEL USING THE
COMBINATION OF CHINESE POSTMAN PROBLEM, TRAVELLING SALESMAN
PROBLEM AND ASSIGNMENT PROBLEM

Priyanka Bhalerao^{1*} & Seema Bagora²
^{*1&2}Shri Vaishnav Vidyapeeth Vishvavidyalaya, Indore

ABSTRACT

Graph theory is the study of the properties and applications of graphs. This particular field of mathematics studies and finds solutions by the study of graphs that are mathematical structures. Mathematical modeling is an activity in which we make mathematical structures to describe the behavior of various activities of our interest in many ways using words, drawing or sketches, computer programs, mathematical formulae etc. We are proposing a business logistics model where the distance travelled by goods and merchandise is minimized along with the allotment of minimum time for each activity we are using the combination of Travelling Salesman Problem, Chinese Postman Problem and Assignment Model to create this least time least distance model. In order to create this solutions, we evaluate the individual solutions of the above stated problems and make a combinatorial structure which represents a real life situation of a Logistics Industry to create a mathematical model for assigning a least distance least time model.

Keywords- *Mathematical modeling, Chinese Postman Problem, Travelling Salesman Problem, Assignment Problem, Logistics.*

I. INTRODUCTION

Graph theory

Graph theory is the study of the properties and applications of graphs. This particular field of mathematics studies and finds solutions by the study of graphs that are mathematical structures. These structures are used to model pairwise relationships between objects by the method of graphical representations and mathematical derivations. Graph theory is considered to be a very interesting and exciting field of mathematics. It also helps a mathematician to derive extremely useful proofs in the field of discrete mathematics.

Graph labeling:

In the mathematical discipline of graph theory, graph labeling is the assignment of labels traditionally represented by integers to the edges or vertices or both, of a graph.

Mathematical modeling:

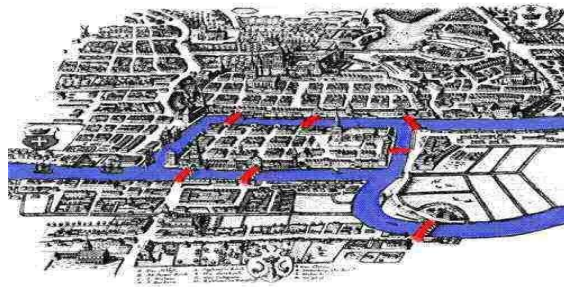
The main aim of mathematical modeling is to make an attempt to study real life problems and break them into parts and convert them into mathematical terms and expression. This also includes developing methods to represent relationships between objects in a real-time physical situation. Mathematical modeling is an activity in which we make model to describe the behavior of various phenomenal activities of our interest in many ways using words, drawing or sketches, computer programs, mathematical formulae etc.

A mathematical model is the mathematical description of a real situation. A model is a mathematical representation of a problem in the form of physical, biological or information system. A mathematical model consists of a set of unproved statements that are called postulates or axioms. In other words, it is a set of undefined terms which are also known as primitives and the set of all related theorems as well as non-related theorems that are deducible from these postulates.

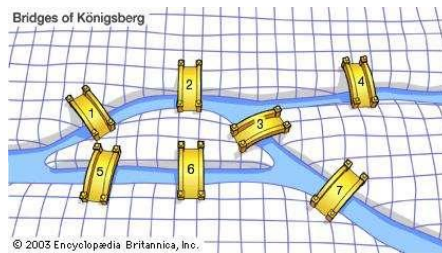
Following are the some mathematical models:

Seven Bridges of Königsberg (in Prussia, 18th century):

The seven bridges of Königsberg is a problem inspired by an actual place and situation. The city of Kaliningard, Russia(at the time Königsberg, Germany) is situated on the Pregolya river, and included two large islands which were connected to each other and the main land by seven bridges. It is historical notable problem in mathematics.

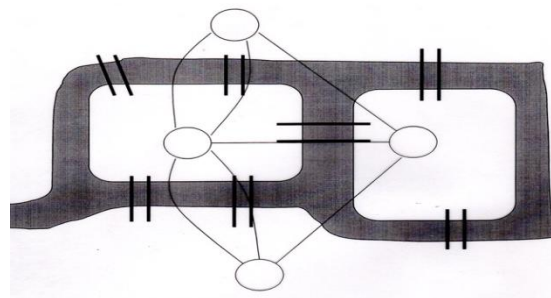


Problem: To find a way or route to cross every bridge exactly once and return to the starting point. One must not cross any bridge more than once.

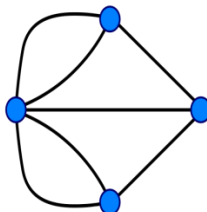


Analysis: Leonhard Euler in 1735 suggested a solution to the Königsberg seven bridges problem.

Graphical Model: To find a path that uses each edge in this undirected graph exactly once. Here edges represent the bridges and the vertices represent the islands.



Solution: Leonhard Euler invented a diagram that he called a network in 1735 which was made of vertices and arcs. These vertices were represented by dots and the arcs by lines. With the help of this network diagram, it is impossible to design such a walk. This was because all the vertices of this network diagram had odd edges. This diagrammatic representation of the network was called as a Graph and is considered as a basic mathematical structure.



Contribution This problem started two important branches of modern mathematics namely graph theory and topology. The problem of seven bridges of Konigsberg is an example of a routing problem like garbage collector who must visit every house once every day but only once.

Graph coloring

Graph coloring is a special case of graph labeling in graph theory. In graph coloring, we assign labels to the elements of a graph which are having a particular set of constraints. These labels are represented by assigning them the typical traditional colors. In simple terms, vertex coloring is the technique by coloring the color the vertices of a graph in a unique way such that no two adjacent vertices share the same color. This holds true for edge coloring where no single pair of edges have the same color of both the edges as well as face coloring where no two edges or no two faces/regions in a planar graph share the same color in-case of a planar graph that share a common boundary.

Problem: Given a geographic map, is it possible to color it with four colors so that any two regions that share a common border are assigned different colors?

This problem can be framed as: “The four color theorem”. The four color theorem was proven in 1967 by Kenneth Appel and Wolfgang Haken.

Model: Any map can be colored with only four colors so that new region having a common boundary have the same color. Those regions that have only a point in common are not considered to have shared boundaries.

Solution: It was the first major theorem to be proved using computer.



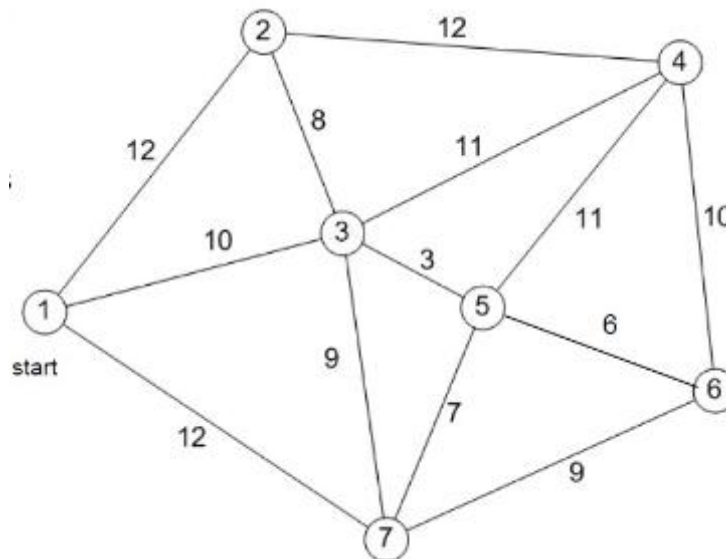
Contribution: we can use minimum number of colors required to color a map so that no two adjacent countries are the same color is four.

II. DISCUSSION

Travelling salesman problem

A salesman must visit every location in a city and return to the starting location. In what order should the salesman visit the location in the city to minimize the distance traveled. In other words, a travelling salesman problem asks for the shortest route to visit a collection of cities and returns to the starting point. British mathematician Thomas Kirkman and Irish mathematician W. R. Hamilton first formulated this problem in 19th century.

Model: This problem can be modeled using a graph where the location on the city is the vertices and the edges are the paths between each location.



Solution: It is a network problem consisting of finding a route from one node to another that yield minimum cost

Definition: In a graph a cycle that passes through every vertex is called a Hamiltonian cycle.

Contribution: This model is very useful for solving routing problems, computer wiring that connected together computer components using minimum wire length and for job sequencing jobs in order to minimize total set up time between jobs. TPS has several applications even in its purest formulation, planning, logistics and the manufacture of microchips.

ASSIGNMENT PROBLEM

In the branch of optimization or operations research in mathematics, one of the most fundamental combinatorial optimization problems is the assignment problem. This method was developed by D. Konig a Hungarian mathematician and is therefore known as the Hungarian method of assignment problem and is a special case of the transportation problem which is a special case of the minimum cost flow problem or we can say where the objective is to minimize the cost or time of completing a number of persons.

Model: This model is related to distribution or allocation of ‘n’ jobs to ‘k’ persons, so that one and only one job is assign to each person to minimize the cost or time of completing the job.

Assignment	Job1	Job2	Job3						Job n
Worker1									
Worker 2									
Worker 3									
Worker k									

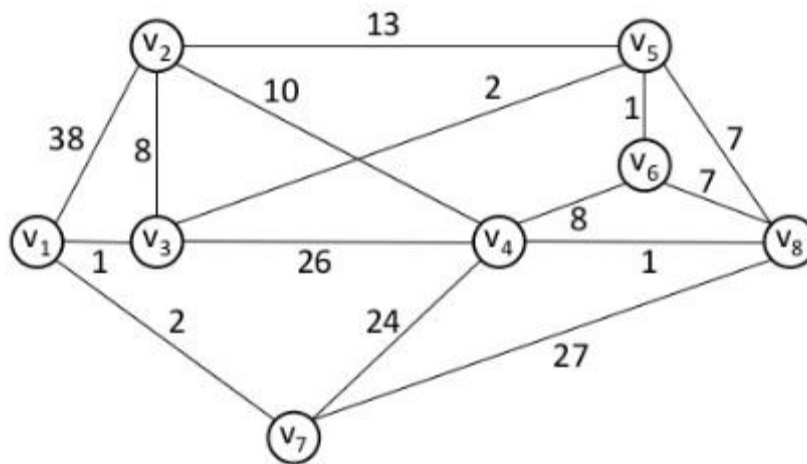
Nowadays assignment problem is not restricted to assigned optimized allocation of work but it is used as indispensable tool for decision making of different problems such as contract to bidders, vehicles to routes, scheduling of airlines, railways, allocation of salesman to different sales territories to maximize sales, classes to

available rooms in a college, even a batting position of a batsman in a cricket match and in many more situations. Some of applications of assignment problem's various diverse business situations areas are:

1. In assigning machines to factory orders.
2. In assigning machines to factory orders.
3. In assigning sales, marketing people to sales territories.
4. In assigning teachers to classes.
5. In assigning accountants to accounts of the clients.

Chinese postman problem (route inspection problem)

Given a weighted graph, this problem deals with finding a route of minimum weight that traverses every edge at least once and returns to the starting vertex. This is called a Postman's problem because a postman needs to traverse every street (edge) in his or her round so that it covers all the houses. Chinese refers to the nationality of the mathematician Mei Ko Kwan-who first considered the problem in 1962 and not to the nationality of the postman.



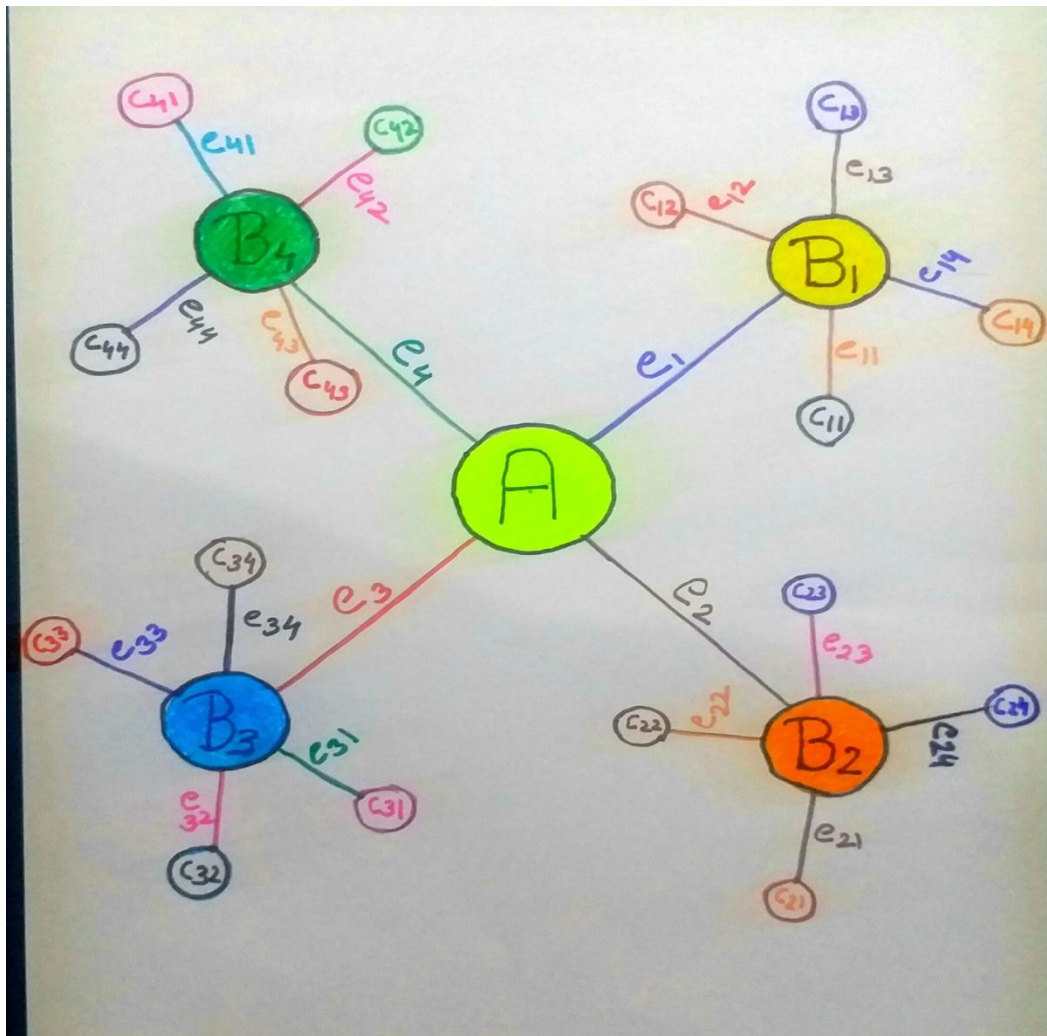
Problem: The logistics networks or process of goods distribution at different stages. The world logistic has originated from Greek word 'logistikes' and the Latin word 'logisticus' which means science of computing and calculating.

Logistics management is the process of planning, implementation and controlling the flow and storage of goods, services and related information such as data and transit details from the origin point to the consumption point in the most effective and efficient way in order to solve the need of the requirements of the customer.

Model: A model of distribution process which shows the distribution of goods at various stages and various problems that arise during distribution.

Solution: To make distribution proper is to remove the problems which arise during this process such as lack of communication, transportation problem, lack of responsibility by agents, lack of networks, lack of coordination etc. Now to overcome all this problems we need to create an algorithm for coordination by the help of various types of graph labeling.

We are proposing a business logistics model where the distance travelled by goods and merchandise is minimized along with the allotment of minimum time for each activity we are using the combination of travelling salesman problem, Chinese postman problem and assignment model to create this least time least distance model.



Let $e \in E$ where the above structure represents a combinatorial structure of the sales network of a company.
Let A be the Central Office (Head Office) of a company. Let it have n Regional Offices and every Regional Office has k Area Offices under it.

Let $B_1, B_2, B_3, \dots, B_n$ be the Regional Offices of the company.

Let the Area Offices be

$C_{n1}, C_{n2}, C_{n3}, \dots, C_{nk} \quad C_{(n+1), 1}, C_{(n+1), 2}, C_{(n+1), 3}, \dots, C_{(n+1), k} \quad C_{(n+2), 1}, C_{(n+2), 2}, C_{(n+2), 3}, \dots, C_{(n+3), k}$

For proper sales management of the organization, following are the set of conditions that need to be followed for the travelling of each Sales Manager.

- 1) Let Head Sales Manager has to travel from A (Head Office) to $B_1, B_2, B_3, \dots, B_n$ (regional offices) once a month.
- 2) Similarly, let each Regional Sales Manager stationed at Regional Office has to visit every Area Office and report it to Head Office once a month.
- 3) Let Head-Sales Manager has to meet every Regional Sales Manager once a month and every Area Sales Manager has to meet his respective Regional Sales Manager once a month.

- 4) The meeting of a lower manager to his respective higher manager has to happen only after he has met his lower manager for the purpose of reporting.
- 5) Every lower manager also has to report to his immediate upper Office once a month for reporting.

III. CONCLUSION

The above problem of Graphical Modeling takes into account three basic Graphical Solution Models that are:

- a) Chinese Postman Problem (Route Inspection Problem)
- b) Travelling Salesman Problem
- c) Assignment Problem for a Weighted Bipartite Graph

The solution of the above drawn Logistical Model will assign each type of manager the least number of trips travelling least distance so as there is no overlap and repeat meetings

REFERENCES

- [1] Harry F. and Norman R.Z, “Graph theory as a mathematical model in social science” Research center for Growth Dynamics, University of Michigan.
- [2] Bovelas. A “A mathematical model for group structure”. *Applications Anthropol*, 1948, 7, 16-30.
- [3] Tanveer S. “Application of Graph theory in Representing and Modeling Traffic Control Problems” *International Journal of Mathematics and Computer Applications. Research (IJMCAR) ISSN(P):2249-6966;ISSN(E),2249-8060.Vol. 6, June 2016, 29-34.*
- [4] “Konigberg Bridges” *The Mac Tutor History of mathematics Archive.*
- [5] Lawson D. and Marion G, “An Introduction to Mathematics Modeling” *Seminar on Bioinformatics and Statistics, Scotland 2008*
- [6] Giordano, Fox, Houston, Weir, “Math/stat 2003 modeling graph theory (march 23/25)from text *A First Course in Mathematical Modeling, 2009.*
- [7] Sanad A, Hasni, I.S. K.G. Subramanian, “Some application of Eulerian Graph School of Mathematics Science Education Technomathematics Research Foundation vol.2 No2,1-10,2009
- [8] Estrada E. “Graph and Network Theory” *Department of Mathematics and Statistics, University of Strathclyde, Glasgow.*